Section 5.3
Definite Integrals and Antiderivatives

Rules For Definite Integrals

1. *Order of Integration:* $\int_b^a f(x)\,dx = -\int_a^b f(x)\,dx$

2. *Zero:* $\int_a^a f(x)\,dx = 0$

3. *Constant Multiple:* $\int_a^b kf(x)\,dx = k\int_a^b f(x)\,dx$

$$\int_a^b -f(x)\,dx = -\int_a^b f(x)\,dx \qquad k = -1$$

4. *Sum and Difference:* $\displaystyle\int_a^b (f(x) \pm g(x))\,dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$

5. *Additivity:* $\displaystyle\int_a^b f(x)\,dx + \int_b^c f(x)\,dx = \int_a^c f(x)\,dx$

6. *Max-Min Inequality:* If max f and min f are the maximum and minimum values of f on $[a, b]$, then

$$\min f \cdot (b - a) \le \int_a^b f(x)\,dx \le \max f \cdot (b - a).$$

7. *Domination:* $f(x) \ge g(x)$ on $[a, b] \Rightarrow \displaystyle\int_a^b f(x)\,dx \ge \int_a^b g(x)\,dx$

$$f(x) \ge 0 \text{ on } [a, b] \Rightarrow \int_a^b f(x)\,dx \ge 0$$

Using the Rules for Definite Integrals

Suppose

$$\int_{-1}^{1} f(x)\,dx = 5, \quad \int_{1}^{4} f(x)\,dx = -2, \quad \text{and} \quad \int_{-1}^{1} h(x)\,dx = 7.$$

-1 4

Find each of the following integrals, if possible.

(a) $\int_{4}^{1} f(x)\,dx = 2$ (b) $\int_{-1}^{4} f(x)\,dx = 3$ (c) $\int_{-1}^{1} [2f(x) + 3h(x)]\,dx$ $= 31$

(d) $\int_{0}^{1} f(x)\,dx$ Not enough info.

(e) $\int_{-2}^{2} h(x)\,dx$ N.E.I.

(f) $\int_{-1}^{4} [f(x) + h(x)]\,dx$ N.E.I.

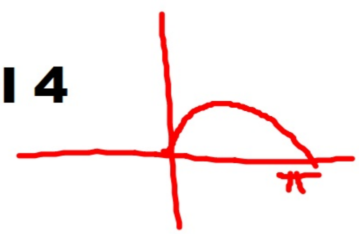
Show that the value of $\int_0^1 \sqrt{1 + \cos x}\, dx$ is less than 3/2.

$$\int_0^1 \sqrt{1+\cos x}\, dx \le \max \sqrt{1+\cos x} \cdot (b-a$$

$$\int_0^1 \sqrt{1+\cos x}\, dx \le \sqrt{2} \cdot (1-0)$$

$$\int_0^1 \sqrt{1+\cos x}\, dx \le \sqrt{2} <$$

Show that $\displaystyle\int_0^{\pi} \sin x \, dx$ **cannot equal 4**



$$\min f \cdot (b-a) \le \int_0^{\pi} \sin x \, dx \le \max f \, (b-a)$$

$$0 \cdot (\pi - 0)$$

$$0 \le \int_0^{\pi} \sin x \, dx \le 1 \, \pi$$

Average (Mean) Value

If f is integrable on [a,b], its average (mean) value on [a,b] is

$$av(f) = \frac{1}{b - a}\int_a^b f(x)\,dx$$

Average y value.......

NOT average rate of change!!!

Find the average value of f(x) = 4 - x^2 on [0,3]. Does f actually take on this value at some point in the given interval?

$$Av = \frac{1}{3-0}\int_0^3 (4-x^2)\,dx$$

$$= \frac{1}{3}(3) = 1$$

$$4-x^2=1$$

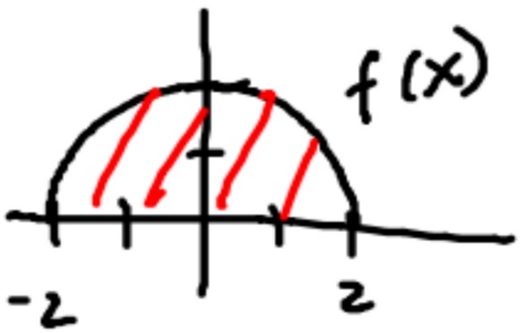
$$-x^2=-3$$

$$x^2=3$$

$$x=\pm\sqrt{3}$$

@ $X=\sqrt{3}$

Find the average value of f(x) on [-2,2]



$$\frac{1}{2}\pi r^2$$

$$\frac{1}{2}\pi(4)$$

$$2\pi$$

$$\frac{1}{2-(-2)}\int_{-2}^{2} f(x)\,dx$$

$$\frac{1}{4}(2\pi) = \frac{2\pi}{4}$$

$$= \frac{\pi}{2}$$

Find the average value of $f(x) = 3x^3 - x^2 + 1$ on $[-1,7]$

$$\frac{1}{7-(-1)}\int_{-1}^{7} 3x^3 - x^2 + 1 \, dx$$

$$\frac{1}{8}\left(1693\tfrac{1}{3}\right) = 211.67$$

Mean Value Theorem for Integrals

If f is continuous on [a,b] then there is a c in [a,b] such that

$$f(c) = \frac{1}{b - a} \int_a^b f(x)\, dx$$

$$\frac{d}{dx} \int_a^x f(t)\, dt = f(x)$$

The integral is an antiderivative of f

If F is any antiderivative of f, then

$$\int_a^x f(t)\, dt = F(x) + C$$

for some constant C

Substituting a in for x gives the following

$$\int_a^a f(t)\,dt = F(a) + C$$

$$0 = F(a) + C$$

$$C = -F(a)$$

Therefore $$\int_a^x f(t)\,dt = F(x) - F(a)$$

Find $\displaystyle\int_{0}^{\pi} \sin x \, dx$ using the formula

$$\int_{a}^{x} f(t)\, dt = F(x) - F(a)$$